15. $f(x) = \int_1^{x^2} \sqrt{t^2+3}\, dt$

$f'(2) = \sqrt{2^4+3} \cdot 2 \cdot 2 = \sqrt{19} \cdot 4$

$f'(x) = \sqrt{x^4+3} \cdot 2x$

$\frac{d}{dx} \int_a^x f(t)\,dt = f(x)$

$$\boxed{\frac{d}{dx} \int_a^{g(x)} f(t)\,dt = f(g(x))\, g'(x)}$$

Mar 9-7:25 AM

21 $\frac{dy}{dt} = my$

$\int \frac{dy}{y} = \int m\, dt$ sep. var.

$\ln|y| = mt + c$

$y = e^{mt+c}$

$y = e^{mt} \cdot e^c$

$y = Ae^{mt}$

$$\boxed{\begin{array}{l} \frac{dy}{dt} = ky \\ y = y_0 e^{kt} \end{array}}$$

Mar 9-8:00 AM

22. $f(x) = -x^6 + x^3 - 2$

f decreasing $f' < 0$

$f'(x) = -6x^5 + 3x^2 < 0$

$3x^2(-2x^3 + 1) < 0$

$-2x^3 + 1 < 0$

$-2x^3 < -1$

$x^3 > \frac{1}{2}$

$x > \sqrt[3]{\frac{1}{2}}$

$\left(\sqrt[3]{\frac{1}{2}}, \infty\right)$

Mar 9-8:06 AM

24. $0 \le t \le 4$ $v = t^3 - 4t^2 - 3t + 2$

min $\rightarrow$ $a = 3t^2 - 8t - 3$

$a' = 6t - 8 = 0$

$t = \frac{8}{6} = \frac{4}{3}$

$a'' = 6 > 0$

$a\left(\frac{4}{3}\right) = 3\left(\frac{4}{3}\right)^2 - 8\left(\frac{4}{3}\right) - 3$

$= 3 \cdot \frac{16}{9} - 8 \cdot \frac{4}{3} \cdot \frac{3}{3} - 3 \cdot \frac{9}{9}$

$= \frac{48 - 96 - 27}{9} = \frac{-75}{9}$

$= \frac{-25}{3}$

Mar 9-8:11 AM

27) ave value $y = x^3\sqrt{x^4+9}$

$[0,2]$

$$\boxed{\text{ave value} = \bar{y} = \frac{1}{b-a}\int_a^b f(x)\,dx}$$

$$\frac{1}{2-0}\int_0^2 x^3\sqrt{x^4+9}\,dx$$

let $u = x^4+9$

$\frac{du}{dx} = 4x^3$

$du = 4x^3\,dx$

$\frac{du}{4x^3} = dx$

$$\int \cancel{x^3}\sqrt{u}\,\frac{du}{4\cancel{x^3}} = \frac{2}{3}u^{3/2}\cdot\frac{1}{4}$$

$$\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{2}{3}(x^4+9)^{3/2}\Big|_0^2$$

$$\frac{1}{12}\left(25^{3/2} - 9^{3/2}\right)$$

$$\frac{1}{12}\left(5^3 - 3^3\right)$$

$$\frac{1}{12}(125-27)$$

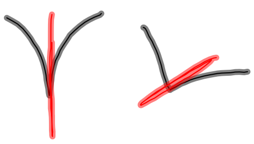
$$\frac{1}{12}(98) =$$

$$\frac{98}{12} = \frac{49}{6}$$

Mar 9-8:15 AM

Review 5 Differentiability

not diff: cusp, jump, hole, "pointy", vert tan, vert. asymptote



if diff then cont

if cont then diff ??

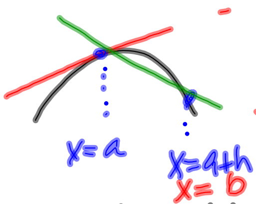
No, heck no

Mar 9-8:23 AM

f is diff.

looks like?

smooth



tan line exists

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{b \to a} \frac{f(b) - f(a)}{b - a}$$

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f is diff

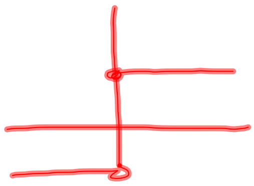
f' is cont

if f' is cont then f is diff

if f is diff then f' is cont?

f'

?



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